Chem. 550
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CLASSICAL MECHANICS PROBLEM 1

A particle in one dimension is experiencing a harmonic potential

$$V(x) = \frac{1}{2} m\omega^2 x^2 \ .$$

a) Write the Lagrangian function for this system.

b) Write the Euler-Lagrange equation of motion.

c) Transform the Lagrangian to obtain the Hamiltonian function.

d) Write Hamilton's equations of motion.

e) Show that the Euler-Lagrange equations are equivalent to Hamilton's equations.

f) These equations have two linearly independent solutions, which are sine and cosine functions. Show that the function

$$x(t) = x_0 \cos\omega t + \frac{p_0}{m\omega} \sin\omega t \ ,$$

along with an expression for $p(t)$ which you should obtain, satisfies Hamilton's equations of motion for the initial conditions x_0, p_0.