$$|\phi_1\rangle = \frac{1}{\sqrt{2}}(|R\rangle + |L\rangle), \qquad |\phi_2\rangle = \frac{1}{\sqrt{2}}(|R\rangle - |L\rangle)$$

$$E_1 = -\hbar\Omega \qquad E_2 = +\hbar\Omega$$

$$|R\rangle = \frac{1}{\sqrt{2}}(|\phi_1\rangle + |\phi_2\rangle), \qquad |L\rangle = \frac{1}{\sqrt{2}}(|\phi_1\rangle - |\phi_2\rangle)$$

(a) $|\psi_0\rangle = C_R|R\rangle + C_L|L\rangle, \qquad C_R = \langle R|\psi_0\rangle, \quad C_L = \langle L|\psi_0\rangle$

$$= C_R \frac{1}{\sqrt{2}}(|\phi_1\rangle + |\phi_2\rangle) + C_L \frac{1}{\sqrt{2}}(|\phi_1\rangle - |\phi_2\rangle)$$

$$= \frac{1}{\sqrt{2}}(C_R + C_L)|\phi_1\rangle + \frac{1}{\sqrt{2}}(C_R - C_L)|\phi_2\rangle$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(C_R + C_L)e^{i\Omega t}|\phi_1\rangle + \frac{1}{\sqrt{2}}(C_R - C_L)e^{-i\Omega t}|\phi_2\rangle$$

$$= \frac{1}{2}(C_R + C_L)e^{i\Omega t}(|R\rangle + |L\rangle) + \frac{1}{2}(C_R - C_L)e^{-i\Omega t}(|R\rangle - |L\rangle)$$

$$= (C_R \cos\Omega t + i C_L \sin\Omega t)|R\rangle + (i C_R \sin\Omega t + C_L \cos\Omega t)|L\rangle$$

$$C(t) = \langle\psi_0|\psi(t)\rangle$$

$$= (C_R\langle R| + C_L\langle L|)\Big[(C_R\cos\Omega t + i C_L \sin\Omega t)|R\rangle$$

$$+ (i C_R \sin\Omega t + C_L\cos\Omega t)|L\rangle\Big]$$

$$= (C_R^2\cos\Omega t + i C_R C_L \sin\Omega t) + (i C_R C_L \sin\Omega t + C_L^2 \cos\Omega t)$$

$$= \cos\Omega t + 2 i C_R C_L \sin\Omega t$$

(b) For $|\psi_0\rangle = |\phi_1\rangle$, $C_R = \frac{1}{\sqrt{2}} = C_L$. Then

$$C(t) = \cos\Omega t + i\sin\Omega t = e^{i\Omega t} = \langle\phi_1|e^{-i\hat{H}t/\hbar}|\phi_1\rangle$$

so we recover the known evolution of an eigenstate

(c) For $|\psi_0\rangle = |R\rangle$, $c_R = 1$, $c_L = 0$

$$|\psi(t)\rangle = \cos\Omega t\, |R\rangle + i\sin\Omega t\, |L\rangle$$

$C(t) = \cos\Omega t$, $P(t) = |\langle\psi_0|\psi(t)\rangle|^2 = \cos^2\Omega t$. With $\tau = \frac{\pi}{\Omega}$,

$t = \frac{1}{4}\tau$: $\cos\Omega t = \frac{\sqrt{2}}{2} = \sin\Omega t$, so

$|\psi(\frac{1}{4}\tau)\rangle = \frac{1}{\sqrt{2}}|R\rangle + \frac{i}{\sqrt{2}}|L\rangle$, $C(\frac{1}{4}\tau) = \frac{1}{\sqrt{2}}$, $P(\frac{1}{4}\tau) = \frac{1}{2}$.

$t = \frac{1}{2}\tau$: $\cos\Omega t = 0$, $\sin\Omega t = 1$ so

$|\psi(\frac{1}{2}\tau)\rangle = i|L\rangle$, $C(\frac{1}{2}\tau) = 0$, $P(\frac{1}{2}\tau) = 0$.

$t = \frac{3}{4}\tau$: $\cos\Omega t = -\frac{\sqrt{2}}{2}$, $\sin\Omega t = \frac{\sqrt{2}}{2}$, so

$|\psi(\frac{3}{4}\tau)\rangle = -\frac{1}{\sqrt{2}}|R\rangle + \frac{i}{\sqrt{2}}|L\rangle$, $C(\frac{3}{4}\tau) = -\frac{\sqrt{2}}{2}$, $P(\frac{3}{4}\tau) = \frac{1}{2}$

$t = \tau$: $\cos\Omega t = -1$, $\sin\Omega t = 0$ so

$|\psi(\tau)\rangle = -|R\rangle$, $C(\tau) = -1$, $P(\tau) = 1$.

$t = \frac{5}{4}\tau$: $\cos\Omega t = -\frac{\sqrt{2}}{2} = \sin\Omega t$ so

$|\psi(\frac{5}{4}\tau)\rangle = -\frac{1}{\sqrt{2}}|R\rangle - \frac{i}{\sqrt{2}}|L\rangle$, $C(\frac{5}{4}\tau) = -\frac{1}{\sqrt{2}}$, $P(\frac{5}{4}\tau) = \frac{1}{2}$.

$t = \frac{3}{2}\tau$: $\cos\Omega t = 0$, $\sin\Omega t = -1$, so

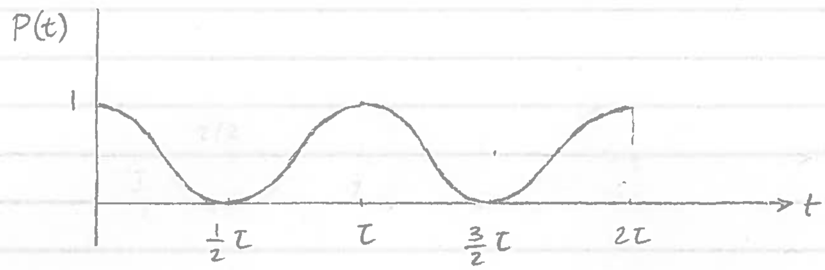
$|\psi(\frac{3}{2}\tau)\rangle = -i|L\rangle$, $C(\frac{3}{2}\tau) = 0$, $P(\frac{3}{2}\tau) = 0$.

$t = \frac{7}{4}\tau$: $\cos\Omega t = \frac{\sqrt{2}}{2}$, $\sin\Omega t = -\frac{\sqrt{2}}{2}$, so

$|\psi(\frac{7}{4}\tau)\rangle = \frac{1}{\sqrt{2}}|R\rangle - \frac{i}{\sqrt{2}}|L\rangle$, $C(\frac{7}{4}\tau) = \frac{\sqrt{2}}{2}$, $P(\frac{7}{4}\tau) = \frac{1}{2}$.

$t = 2\tau$: $\cos\Omega t = 1$, $\sin\Omega t = 0$ so

$|\psi(2\tau)\rangle = |R\rangle$, $C(2\tau) = 1$, $P(2\tau) = 1$.



(d) $|\psi_0\rangle = \frac{1}{2}|R\rangle + C_L|L\rangle$. $C_L = \frac{\sqrt{3}}{2}$ (choose $C_L > 0$.)

$$C(t) = \cos\Omega t + \frac{\sqrt{3}}{2}\, i \sin\Omega t$$

$$P(t) = |C(t)|^2 = \cos^2\Omega t + \frac{3}{4}\sin^2\Omega t = 1 - \frac{1}{4}\sin^2\Omega t$$

$$P(\tfrac{1}{4}\tau) = 1 - \frac{1}{4}\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{7}{8}$$

$$P(\tfrac{1}{2}\tau) = \frac{3}{4} \qquad P(\tfrac{3}{4}\tau) = \frac{7}{8} \qquad P(\tau) = 1$$

$$P(\tfrac{5}{4}\tau) = \frac{7}{8}, \quad P(\tfrac{3}{2}\tau) = \frac{3}{4}, \quad P(\tfrac{7}{4}\tau) = \frac{7}{8}, \quad P(2\tau) = 1.$$