

Evol. Models Problem 2

$$\text{In } 3d, \quad \hat{H} = \frac{\hat{P}_x^2}{2m} + \frac{\hat{P}_y^2}{2m} + \frac{\hat{P}_z^2}{2m}$$

Free particle propagator:

$$\langle x_2 y_2 z_2 | e^{-\frac{i}{\hbar} \left(\frac{\hat{P}_x^2}{2m} + \frac{\hat{P}_y^2}{2m} + \frac{\hat{P}_z^2}{2m} \right) (t_2 - t_1)} | x_1 y_1 z_1 \rangle$$

$$= \langle x_2 | e^{-\frac{i}{\hbar} \frac{\hat{P}_x^2}{2m} (t_2 - t_1)} | x_1 \rangle \langle y_2 | e^{-\frac{i}{\hbar} \frac{\hat{P}_y^2}{2m} (t_2 - t_1)} | y_1 \rangle$$

$$\cdot \langle z_2 | e^{-\frac{i}{\hbar} \frac{\hat{P}_z^2}{2m} (t_2 - t_1)} | z_1 \rangle$$

$$= \left(\frac{m}{2\pi i \hbar (t_2 - t_1)} \right)^{3/2} \exp \left\{ \frac{i}{\hbar} \frac{m}{2(t_2 - t_1)} \left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \right] \right\}$$

Note that this can also be written as

$$\left(\frac{m}{2\pi i \hbar (t_2 - t_1)} \right)^{3/2} \exp \left\{ \frac{i}{\hbar} \frac{m}{2(t_2 - t_1)} |\vec{r}_2 - \vec{r}_1|^2 \right\}$$

where $\vec{r} \equiv (x, y, z)$.