Problem 2

$$
\begin{aligned}
\langle \psi_2(t) | \psi_1(t) \rangle &= \langle \psi_2(t_0) | U^\dagger(t, t_0) U(t, t_0) | \psi_1(t_0) \rangle \\
&= \langle \psi_2(t_0) | \psi_1(t_0) \rangle \\
\because U^\dagger U &= \mathbb{I} \qquad \text{(unitarity)}
\end{aligned}
$$