Time Evol Problem 2

(a) $|\Psi(0)\rangle = |\phi_0\rangle$ eigenstate of $\hat{H}$, $E_0 = \frac{1}{2}\hbar\omega$.

$$|\Psi(t)\rangle = |\phi_0\rangle e^{-iE_0 t/\hbar} = |\phi_0\rangle e^{-i\omega t/2}$$

$$\langle \Psi(0)|\Psi(t)\rangle = e^{-i\omega t/2}$$ $P(t) = 1$. constant.

(b) $$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|\phi_0\rangle + |\phi_1\rangle\right)$$

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left(|\phi_0\rangle e^{-i\omega t/2} + |\phi_1\rangle e^{-3i\omega t/2}\right)$$

$$\langle \Psi(0)|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left(\langle\phi_0| + \langle\phi_1|\right)\frac{1}{\sqrt{2}}\left(|\phi_0\rangle e^{-i\omega t/2} + |\phi_1\rangle e^{-3i\omega t/2}\right)$$
$$= \frac{1}{2}\left(e^{-i\omega t/2} + e^{-3i\omega t/2}\right) = \frac{1}{2}e^{-i\omega t/2}\left(1 + e^{-i\omega t}\right)$$

$$P(t) = \frac{1}{4}\left(1+e^{-i\omega t}\right)\left(1+e^{i\omega t}\right) = \frac{1}{4}\left(1 + e^{i\omega t} + e^{-i\omega t} + 1\right)$$

$$P(t) = \frac{1}{2}\left(1+\cos\omega t\right)$$

Now the survival probability oscillates with time