Normal Modes - Prob 2

$$V(r) = \frac{1}{2} k \left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \right]$$

$$K_{ij} = \frac{\partial^2 V}{\partial r_i \partial r_j}$$

$$\therefore \mathbb{K} = \begin{pmatrix} \frac{k}{m_1} & \frac{-k}{\sqrt{m_1 m_2}} & 0 & 0 & 0 & 0 \\ \frac{-k}{\sqrt{m_1 m_2}} & \frac{k}{m_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{k}{m_1} & \frac{-k}{\sqrt{m_1 m_2}} & 0 & 0 \\ 0 & 0 & \frac{-k}{\sqrt{m_1 m_2}} & \frac{k}{m_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{k}{m_1} & \frac{-k}{\sqrt{m_1 m_2}} \\ 0 & 0 & 0 & 0 & \frac{-k}{\sqrt{m_1 m_2}} & \frac{k}{m_2} \end{pmatrix}$$

Since this is a block-diagonal matrix, the eigenvectors & eigenvalues of each block are independent of one another.

$\therefore$ For each block,
$$\begin{vmatrix} \frac{k}{m_1} - \lambda & \frac{-k}{\sqrt{m_1 m_2}} \\ \frac{-k}{\sqrt{m_1 m_2}} & \frac{k}{m_2} - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 - k \left[\frac{1}{m_1} + \frac{1}{m_2} \right] \lambda = 0$$

$$\Rightarrow \lambda \left[\lambda - \frac{k}{\mu} \right] = 0$$

$$\therefore \lambda = 0, \quad \lambda = \frac{k}{\mu}$$

$\therefore$ $\omega_1, \omega_2, \omega_3 = 0 \rightarrow$ Translational

$\omega_4, \omega_5, \omega_6 = \sqrt{k/\mu} \rightarrow$ Sym. stretch