Density matrix – Problem 1

Consider a two-state system, for which the eigenstates of the Hamiltonian are

$$|\Phi_1\rangle = \frac{1}{\sqrt{2}}(|L\rangle + |R\rangle), \quad |\Phi_2\rangle = \frac{1}{\sqrt{2}}(|L\rangle - |R\rangle)$$

where $|L\rangle, |R\rangle$ are orthonormal left- and right-localized states. We define the density operator

$$\hat{\rho} = \frac{1}{2}(|R\rangle\langle R| + |L\rangle\langle L|) + \frac{1}{4}(|R\rangle\langle L| + |L\rangle\langle R|).$$

(a) Write the matrix representation of ρ and show that ρ is normalized.
(b) Express ρ in the eigenstate basis and show that the new matrix is again normalized. What are the populations of the two eigenstates?
(c) Does this density operator correspond to a pure or a mixed ensemble?